Assessment of a cracked pipe subject to transient flow by the Monte Carlo method

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\textbf{A B S T R A C T}

The purpose of this work is to assess the risk of failure of cracked pipe due to fluid transient. A mathematical model has been established based on the mass and momentum conservation laws, the system of hyperbolic partial differential equations has been solved by the method of characteristics and a finite difference method to calculate the maximum pressure in the pipe. Afterwards, a finite element method have been used to perform a reliable assessment analysis of a cracked pipe used in water distribution by using Monte Carlo method and failure assessment diagram (FAD) tools to evaluate the safety factor from deterministic and probabilistic view points.

\textbf{Key words:} Monte Carlo method, failure assessment diagram, water hammer, method of characteristics

1. Introduction

The increasing demand of water to meet the needs of different water uses requires an increase in the water distribution networks, which requires implantation of adequate maintenance strategy to avoid the additional costs of maintenance, and an increase the pressure of the service, and this begs the question about the safe operating of the system. Despite the security measures and standardized design methods, there are other inevitable factors that can affect the structure integrity and leads to the failure of water pipelines. This failure may manifested by two cases, either by rupture or leak, and in both cases the consequences are very disastrous, compounded by compromising the health of the population due to water contamination.

The water pipeline failure\cite{1} reason can be assumed to be corrosion pitting, scratches, gouges, and also service loading conditions depend on soil movement, e.g. ground slip, earthquakes or repeated loading due to road traffic.

The presence of cracks in water pipelines is related to many causes e.g. micro-void, inclusion, manufacturing defects. Theses defects grow under mechanical and environment condition and failure occurs when defect has reached the critical size under service condition or under unusual loading condition such as water hammer.
Water hammer is produced by a rapid change of flow velocity in the pipe line that may be caused by sudden valve opening or closure, failure of a pump, mechanical failure of device, rapid change in demand condition, etc. It could result in violent change of the pressure head, which is then propagated in the water pipeline in the form of a fast pressure wave leading to severe damage [2].

In this present work the water hammer effect on pipe failure is described in order to provide a reliable structure integrity and safety method, using Monte Carlo method and failure assessment diagram.

1.1. Theory/calculation methodology

A defect can be detected by non-destructive test [3], and the question is the defect is acceptable or not. In order to answer to this question there are various methods of assessment of defect assessment e.g. R6 Method, BSI PD6493, SINTAP. In this study a failure assessment diagram is used according to the SINTAP procedure with level 1, in order to be able to make decision about the failure risk of the water pipeline, when a water hammer occurs. The random characteristics of the governing parameters in real pipelines have motivated many researchers to develop probabilistic approaches to assess the probability of failure of pipelines. The Monte Carlo simulation method is used to assess the uncertainty to estimates the safety factor of cracked pipe.

The high instantaneous pressure due to water hammer is calculated from the mathematical model of fluid transient, and then the value of maximum pressure is incorporated into finite element code to calculate the stress intensity factor at the vicinity of crack tip. Once the value of stress intensity factor is calculated, it is used to plot the defect assessment point coordinates.

The value of maximum stress can be calculated via thin-walled hollow cylinder assumption $\sigma = \frac{P D}{2t}$ where $P =$ pressure, $D =$ diameter, $t =$ thickness.

2. Failure assessment diagram

2.1. SINTAP procedure

This procedure [4, 5] is a unitary European Community approved programme to assess structure integrity of a have defect, against the level of failure risk. This procedure is based on the failure mechanics principles. The relationship between applied stress $\sigma$ defect size $a$ and toughness is replaced by two parameters corresponding to brittle fracture $K$ ($K_y = 1$, $L_y = 0$) and plastic collapse $L$ ($K_r = 0$, $L_r = L_{max}$). These parameters can be defined as follows:

$$K_r = \frac{K_L}{K_{IC}} \quad (1)$$

$$L_r = \frac{\sigma_u}{R_c} \quad (2)$$

where: $R_c = \frac{1}{2}(\sigma_y + \sigma_u)$

These two variables represent the ratio between the applied value of either stress or stress intensity factor and the resistance parameter of the corresponding magnitude (yield stress or fracture toughness).
2.2. Level 1 investigation

The failure assessment diagram is limited by the failure assessment curve defined by a relation \( K_r = f(L_r) \), the level of analysis allow us to choose the parameters necessary to establish the risk analysis. The level 1 analysis is the minimum recommended level. This level requires the yield strength, the ultimate stress, and the value of fracture toughness of the material. The FAD curve is defined as follows:

For \( 0 \leq L_r \leq 1 \)

\[
K_r = 1 + \frac{L_r^2}{2} \left[ 0.3 + 0.7 e^{-0.5 L_r^2} \right] \tag{3}
\]

where: \( L_r^{\text{max}} = 1 + \left( \frac{150}{\sigma_y} \right)^{2.5} \)

2.3. Assessment diagram

The assessment diagram is plotted in coordinates \( K_r \) and \( L_r \) [6]. Two particular points of this diagram represent successively brittle fracture conditions \( (K_r = 1, L_r = 0) \) and plastic collapse \( (K_r = 0, L_r = 1) \). The curve which defined the assessment diagram encloses between the coordinates a safe domain.

The loading conditions of a structure are represented by a point A of coordinates \( (K_r^*, L_r^*) \). If this point is inside of this domain, this ensures the structure's integrity. If this point \( C \) is on the curve, failure occurs (Fig.1).

3. Water hammer equation

The mathematical formulation of the transient is developed based on the equation of conservation of mass, the conservation equation of momentum, and the equation of thermodynamic behaviour [7], for unsteady pipe flow. The classical theory of water hammer takes into account the effect
of skin (fluid-wall) friction, approximated by Trikha model. The pipe is straight, thin-walled, linearly elastic and of circular cross-section. The two equations, governing velocity $V$ and pressure $P$ are:

$$\frac{\partial V}{\partial x} + \frac{1}{\rho A} \frac{\partial P}{\partial t} = 0 \tag{4}$$

$$\frac{\partial V}{\partial t} + \frac{1}{\rho A} \frac{\partial P}{\partial x} + \frac{4 T_f}{A D} + g \sin \theta = 0 \tag{5}$$

in which:

- $x = \text{axial distance}$
- $\rho = \text{mass density of liquid}$
- $a = \text{liquid (elastic) wave speed}$
- $t = \text{time}$
- $T_f = \text{friction term}$
- $D = \text{internal pipe diameter}$
- $g = \text{gravitational acceleration}$, and
- $\theta = \text{pipe slope}$.

Equations 4 and 5 make a system of partial hyperbolic differential equations which connect the pressure $P$ and the fluid velocity $V$.

3.1 Determination of the shear stress

To model the friction term, we used the Trikha model [8], which relates wall shear stress in transient laminar pipe flow to the instantaneous mean velocity and weighted past velocity changes.

$$T_f(x,t) = -8 D V x, t + \frac{1}{2} \int_0^t \frac{\partial V}{\partial t} W(t_s) ds \tag{6}$$

in which $n_c = \text{fluid kinematic viscosity}$, $W = \text{a weighting function}$, and $s = \text{variable of integration}$.

3.2 Method of resolution

The method used to solve mathematical systems that govern the phenomenon is the method of characteristics (MOC) [9]. This is used to transform the equations of partial derivative equations to total derivatives which are integrated along the characteristic direction of lines. The MOC transformation of Equns 4 and 5 yields the water hammer compatibility equations which are valid along the characteristic lines. The physical meaning of the characteristics lines is propagation path of pressure wave. The compatibility equations, written in a finite-difference form within the staggered grid are shown in Fig.2.

Along the $C^+$ characteristic line ($\Delta x/\Delta t = +a$):

$$H_i^{j+1} - H_i^{j-1} + B \left( Q_i^{j+1} - Q_i^{j-1} \right) + R Q_i^{j-1} \left| Q_i^{j-1} \right| \Delta x = 0 \tag{7}$$

Along the $C^-$ characteristic line ($\Delta x/\Delta t = -a$):
where: \( B = \frac{\partial}{\partial x A} \) and \( R = \int \frac{f}{(2gDA^2)} \)

4. Case study

In this study a tank pipe valve system is considered Fig.3. The pipe has an axial edge defect of length \( a \) subjected to an internal source pressure \( P = 1.688 \) MPa. The cast iron pipe is used with diameter \( D = 450 \) mm and thickness \( t = 8.6 \) mm with Poisson’s ratio \( \nu = 0.28 \). The mechanical properties are defined in Table 1.

5. Finite element modelling

A finite element code called Ansys APDL has been used to modelling the pipe geometry (Fig.4), considered as a plan strain state. According to the symmetry, only on half of the pipe has been considered. 8-node quadrilateral elements has been adapted to meshing the pipe; we have refined the mesh near the crack tip which represents the critical zone of the pipe.

6. Results and discussion

Different \( a/t \) ratios have been computed to find stress intensity factor. The value of \( K_\text{I} \) has been calculated for service pressure \( P = 1.688 \) MPa and for maximum pressure resulting from water hammer, and for a thickness of \( t = 8.6 \) mm.

According to Fig.5 we note that the stress intensity factor increases with increase in service pressure, and more the ratio \( a/t \) of the defect size increase the more the pressure is important. The arise of stress intensity factor value is due to pipe well section reduction by the crack, which lead to stress concentration effect at the vicinity of the crack tip.

Figures 6 and 7 show the calculus of the pair \((K_\text{I}, L_\text{r})\) for various ratios \( a/t \) (0.1, 0.2, 0.3, 0.4, and 0.5) using the SINTAP code for cracked pipe. The assessment points are given on the figures for different ratios crack sizes. The interpolating curve defined for safety factor \( F_s = 2 \), established the limit zone between safety zone and the security zone.

The failure prediction of the pipe due to water hammer may be considered from the safety factor calculation. Conventionally, it is considered the failure is possible to occur if the safety factor is less than two.

Whenever the ratio \( a/t \) increases, the safety factor also increases, which means that the risk of failure of water pipeline increases. According to Fig.8 the structure is reliable if the value of

<table>
<thead>
<tr>
<th>( \sigma_y ) (MPa)</th>
<th>( \sigma_u ) (MPa)</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( K_{ic} ) (MPa.m(^{1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>420</td>
<td>7050</td>
<td>14.90</td>
</tr>
</tbody>
</table>

*Table 1. Mechanical properties of cast iron*
safety factor is greater than two. Also has been found that the more the pressure increases, the more the safety factor decreases.

According to Fig.8 it has been shown that the pressure is more influential on the safety factor than the ratio a/t of crack size.

It has been found that the safety factor is reduced from the value of safety factor \( F_s = 4.9 \) at a pressure of 1.688 MPa to \( F_s = 1.7 \) at a pressure of 4.85 MPa water hammer for the ratio \( a/t = 0.2 \), which proves that the phenomenon of water hammer is dangerous for the integrity of the structure, which can lead to failure.

7. Probabilistic safety factor

From the probabilistic point of view, the failure assessment diagram can take in account the uncertainty of random variable to calculate the safety factor. The material failure curve is a particular case for which the failure probability is equal to unity, because failure is then a certainty. When using a probabilistic approach, each variable is viewed as a probability distribution. Monte Carlo (MC) method that uses to calculate the failure probability integral, the calculated value be interpreted as a mean value in a stochastic experiment. An estimate is therefore given by averaging a suitably large number of independent outcomes of this simulation. The samples parameters are generated by random numbers from a uniform distribution between 0 and 1. This random number can be used to generate a value of the desired random variable with a given distribution. The advantage with MC simulation is that it is robust and easy to implement into a computer program, and, for a sample size when \( N \) tends to infinity, the estimated probability converges to the exact result. Another advantage is that MC simulation works with any distribution of the random variables. There is no restriction on the limit state functions.

In order to allow determination of safety factor
Fig. 5. Variation of stress intensity factor with a/t for P = 1.688 MPa and P = 4.85 MPa.

Fig. 6. FAD for pressure = 1.688 MPa.

Fig. 7. FAD for pressure = 4.85 MPa.

Fig. 8. Safety factor evolution for P = 1.688 MPa and 4.85 MPa.
by Monte Carlo method within the chosen procedure of Fig.9, the following parameters are treated as random parameters and introduce into the failure assessment diagram:

- fracture toughness
- yield strength
- ultimate tensile strength
- defect depth
- maximum pressure

These random parameters are treated as not being correlated with one another. For each parameter, the distribution is chosen according to the usual distributions described Table 3.

Fracture toughness is assumed to have a Weibull distribution. The probability density function has the following form:

$$f(x) = \left(\frac{k}{c}\right)^{k} \frac{x^{k-1}}{c^{k}} \exp\left(-\frac{x}{c}\right)$$

where $k$ is the shape parameter and $c$ is the scale parameter. The mean and variance of the distribution are given by:

$$\lambda^{c}\left(1+\frac{1}{k}\right)$$

$$\sigma = \lambda^{c}\left[\left(1+\frac{2}{k}\right)\right]^{-\frac{k}{2}}\left(1+\frac{1}{k}\right)$$

Yield strength, ultimate tensile strength, and internal pressure can be assumed mainly to have a normal distributions. An exponential distribution generally prevails for defect size analysis. Consequently the probability density function has the following form:

$$f(x) = \lambda \exp(-\lambda x)$$

Using Monte Carlo method, several assessment, 50 points were generated using the characteristic

<table>
<thead>
<tr>
<th>Mechanical property</th>
<th>Yield strength (Re)</th>
<th>Ultimate strength</th>
<th>Circumferential stress</th>
<th>Fracture Toughness</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>300</td>
<td>420</td>
<td>44.16</td>
<td>14.9</td>
<td>0.86 mm</td>
</tr>
<tr>
<td>CV</td>
<td>0.1</td>
<td>0.1</td>
<td>0.44</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>30</td>
<td>42</td>
<td>19.47</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>Normal</td>
<td>Normal</td>
<td>Normal</td>
<td>Weibull</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

Table 3. Mechanical properties of cast iron and used distribution
parameters of the distribution. Any assessment point in a failure assessment diagram is localized by polar coordinates $r$ and $\theta$. The angle $\theta$ is a parameter which represents the belonging of the assessment point to a failure domain and, for this reason, it is called the angle domain. According to the following Table 4, the angle domain indicates the failure type.

Domain angles $\theta_1$ and $\theta_2$ are presented in Fig.10. In the case of a failure curve given by the SINTAP procedure, values of $\theta_1$ and $\theta_2$ are respectively $\theta_1 = 55^\circ$ and $\theta_2 = 22^\circ$. It has been shown that general trend is that the margin of safety on the FAD is minimum in the middle (elastic-plastic) region, slightly higher in the plastic collapse region and maximum in the brittle fracture region. However, this overall trend is complicated by varying degree of scatter in the different regions. For this reason, we have examined the evolution of the safety factor with $\theta$ from a statistical point of view [10].

Using Monte Carlo method, the $K_r$ and $L_r$ coordinates of the assessment points have been computed and reported in domain failure assessment diagram and the associated safety factor is computed Fig.11. It has been shown that the $\theta$ angle is in range of brittle fracture. Note that the value of the safety factor is relatively high because the internal pressure is relatively low. All data are scattered band of range in the $[\mu - 3\sigma, \mu + 3\sigma]$ region of brittle failure. The safety factor distribution is represented with a Weibull distribution. The goodness of fit was tested with Kolmogorov-Smirnov test, the results indicates that the Weibull distribution is significant at 55%. In the domain failure assessment diagrams, a particular assessment point can be is calculated from mean values of all the variable parameters.

The safety factor and the domain angle are computed for $a/t = 0.1$ and $P = 1.668$ MPa. The results are reported in Table 5. It has been found that the deterministic value of safety factor is close to probabilistic value calculated by Monte carlo Method.

<table>
<thead>
<tr>
<th>Pure brittle fracture</th>
<th>Brittle fracture</th>
<th>Elastic-plastic fracture</th>
<th>Plastic collapse</th>
<th>Instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 = 90^\circ$</td>
<td>$0 &gt; \theta \geq q_1$</td>
<td>$q_1 &gt; \theta \geq q_2$</td>
<td>$q_2 &gt; \theta &gt; 90$</td>
<td>$0 = 0^\circ$</td>
</tr>
</tbody>
</table>

Table 4. Failure domain represented by domain angle
8. Conclusion

A numerical model based on mass, momentum conservation laws is developed to simulate the transient flow and predict the maximum pressure value in the pipe. By using the SINTAP failure diagram assessment code we are able to decide rapidly about the acceptability of the crack defect size. Under transient service pressure, the safety factor is distributed randomly according to a Weibull distribution. We can use this diagram as a tool combined with the Monte Carlo method to assess the uncertainty of parameters in order to estimate the safety of the structure, and to minimize the water risk of failure and the associated costs of maintenance.

Table 5. Deterministic and probabilistic values of safety factor and domain angle

<table>
<thead>
<tr>
<th>P = 1.66</th>
<th>Fs D</th>
<th>Fs P</th>
<th>θ D</th>
<th>θ P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>6.4</td>
<td>6.39</td>
<td>45.29</td>
<td>45.44</td>
</tr>
<tr>
<td>0.2</td>
<td>4.9</td>
<td>4.31</td>
<td>56.31</td>
<td>56.3</td>
</tr>
<tr>
<td>0.3</td>
<td>3.26</td>
<td>3.3</td>
<td>67.31</td>
<td>67.09</td>
</tr>
<tr>
<td>0.4</td>
<td>2.16</td>
<td>2.18</td>
<td>74.88</td>
<td>74.84</td>
</tr>
<tr>
<td>0.5</td>
<td>1.06</td>
<td>1.13</td>
<td>82.5</td>
<td>82.11</td>
</tr>
</tbody>
</table>

Conflicts of interest

All authors have no conflicts of interest to declare.

References
